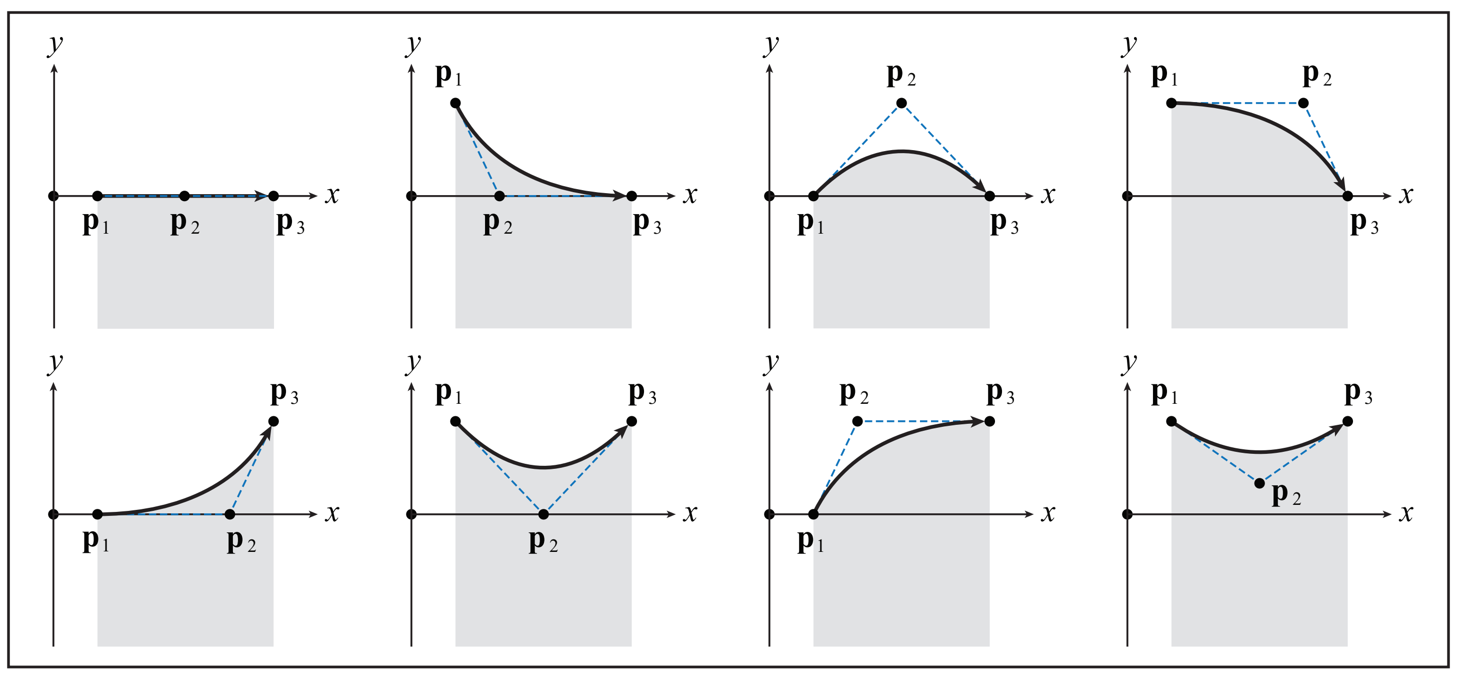
The Slug Algorithm

sluglibrary.com

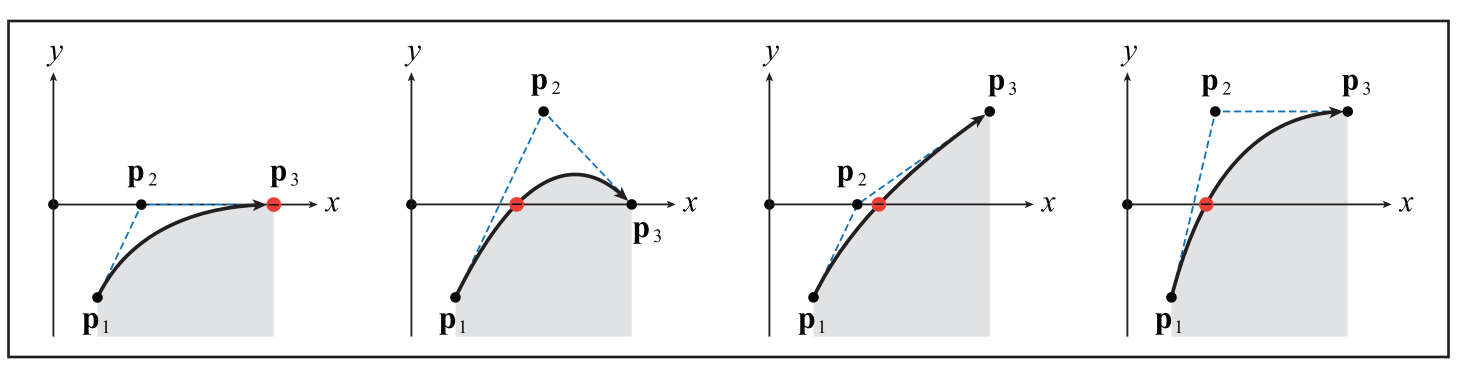
Root Eligibility

Class	$y_3 < 0$	$y_2 < 0$	$y_1 < 0$	Root 2	Root 1
A	0	0	0	0	0
B	0	0	1	1 ⊖	0
C	0	1	0	1 ⊖	1 ⊕
D	0	1	1	1 ⊖	0
E	1	0	0	0	1 ⊕
F	1	0	1	1 ⊖	1 ⊕
G	1	1	0	0	1 ⊕
H	1	1	1	0	0
				0x2E	0x74

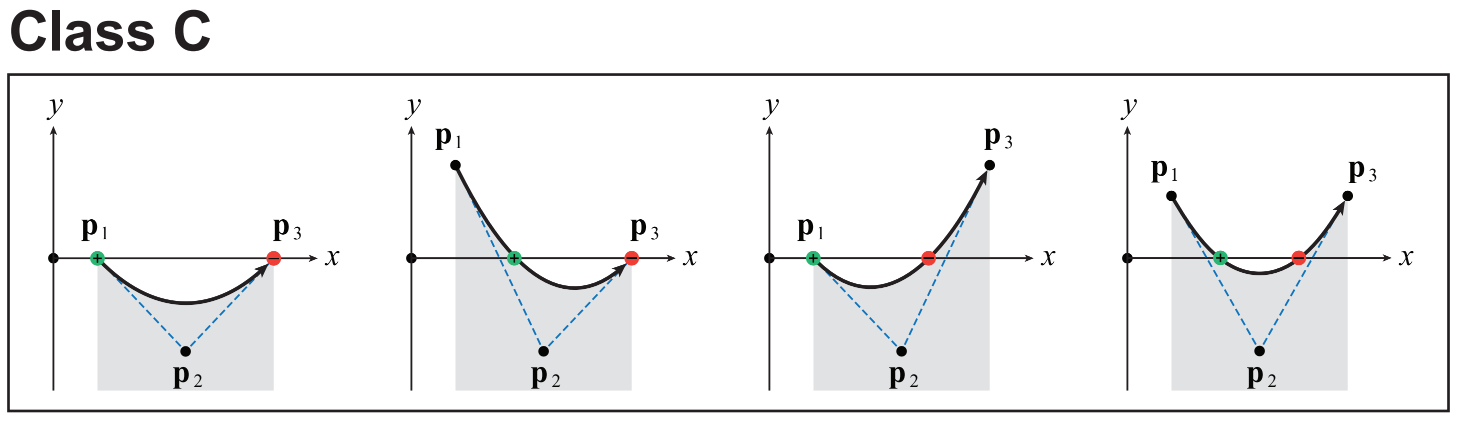
Class A



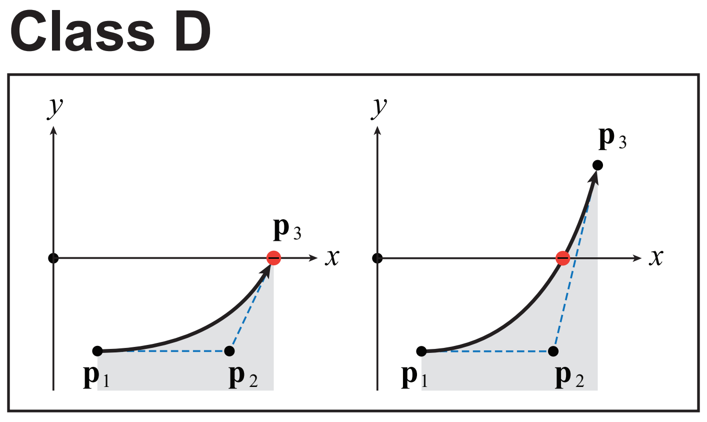
Class B



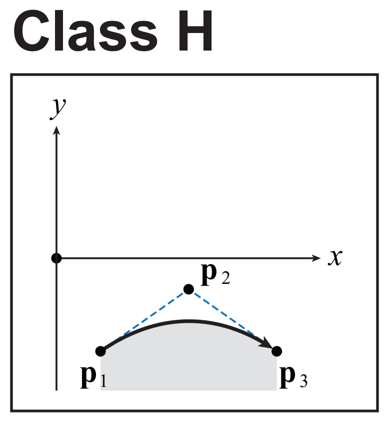
Class C



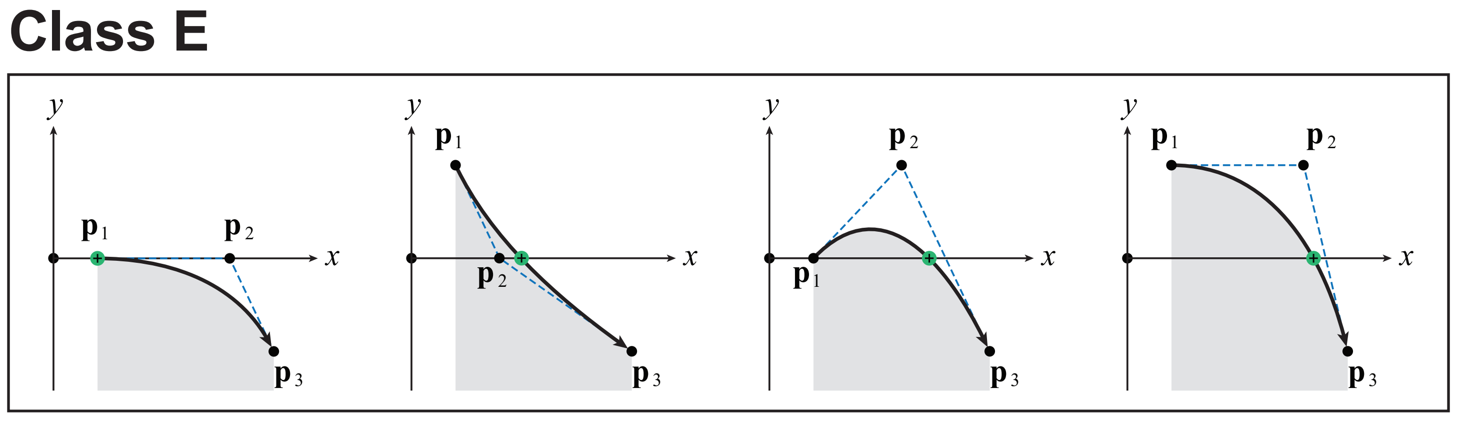
Class D



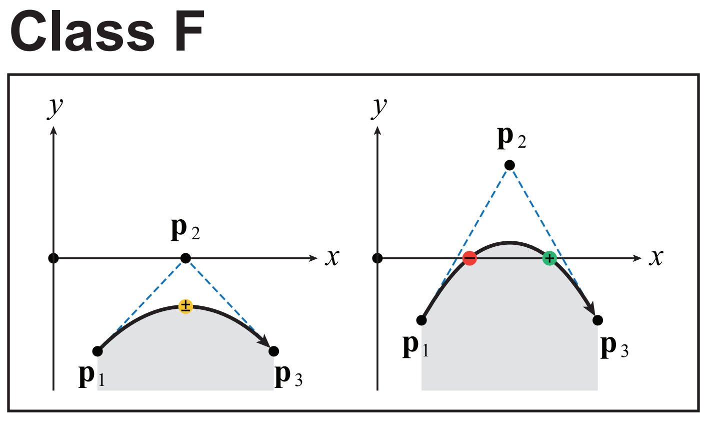
Class H



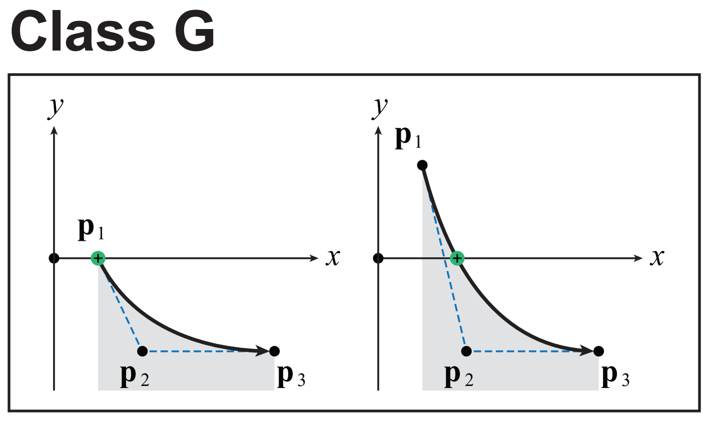
Class E



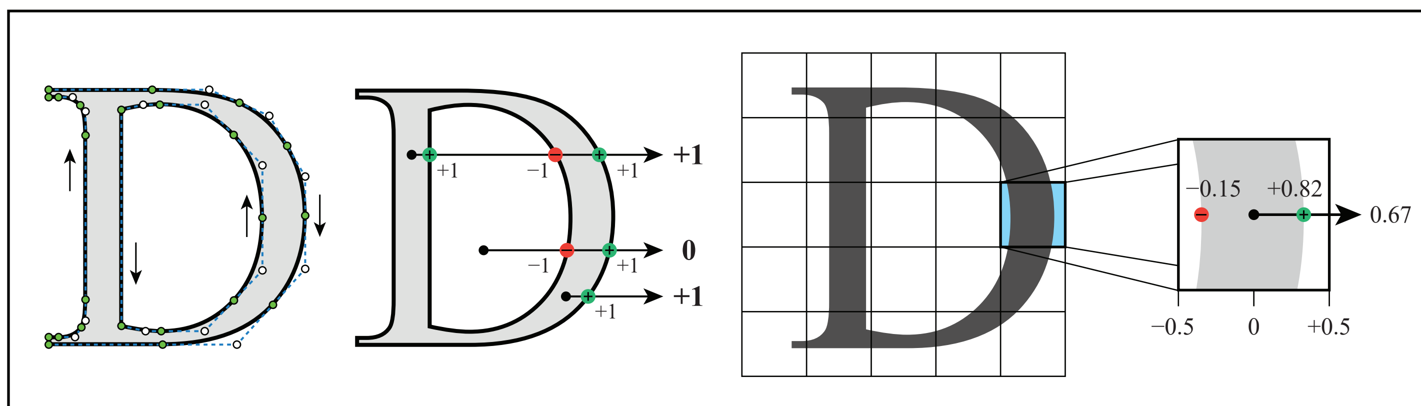
Class F



Class G



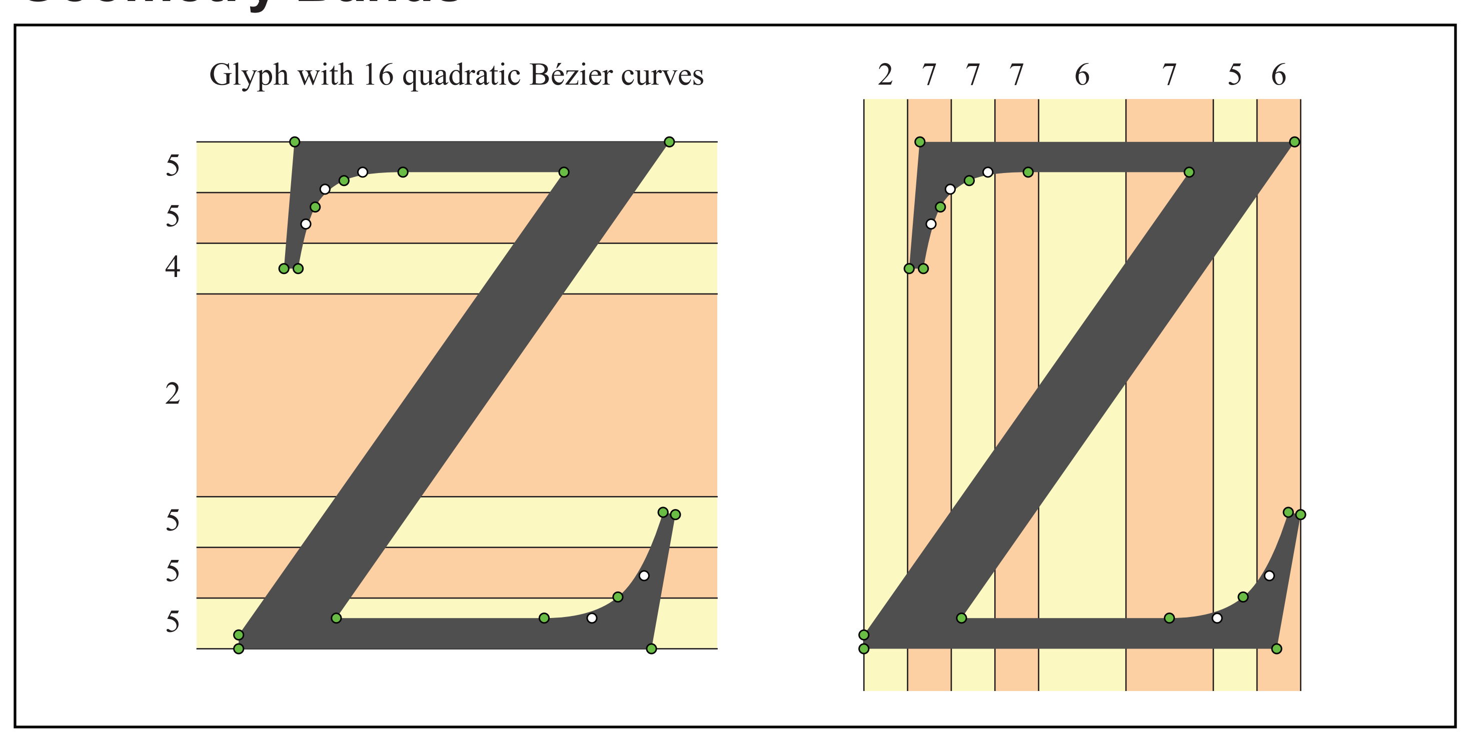
Winding Number



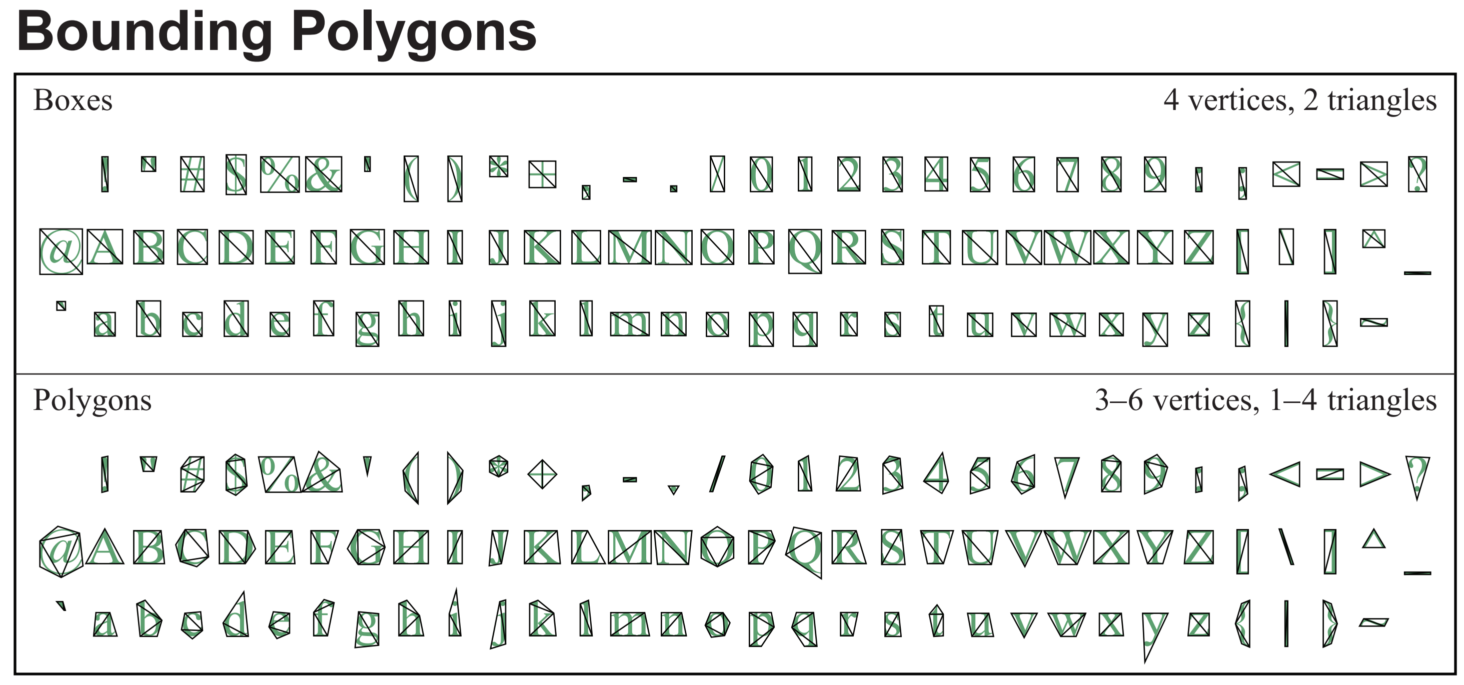
Quadratic Bézier curve	$\mathbf{p}(t) = (1-t)^2\,\mathbf{p}_1 + 2t(1-t)\,\mathbf{p}_2 + t^2\mathbf{p}_3$ $\qquad \mathbf{p}_i = (x_i, y_i)$
Ray intersection equation	$p_y(t) = (y_1 - 2y_2 + y_3)\,t^2 - 2(y_1 - y_2)\,t + y_1 = 0$
Potential solutions	$t_1 = \frac{b - \sqrt{b^2 - ac}}{a}$ $\quad t_2 = \frac{b + \sqrt{b^2 - ac}}{a}$ $\quad \frac{d}{dt}p_y(t_1) \le 0$ $\quad \frac{d}{dt}p_y(t_2) \ge 0$ $a = y_1 - 2y_2 + y_3$ $\quad b = y_1 - y_2$ $\quad c = y_1$
Change to winding number for ray in positive x direction	$+\operatorname{sat}\left(kp_x(t_1) + \frac{1}{2}\right)$ if root 1 eligible ⊕ $-\operatorname{sat}\left(kp_x(t_2) + \frac{1}{2}\right)$ if root 2 eligible ⊖
Change to winding number for ray in negative x direction	$-\operatorname{sat}\left(\frac{1}{2} - kp_x(t_1)\right)$ if root 1 eligible ⊖ $+\operatorname{sat}\left(\frac{1}{2} - kp_x(t_2)\right)$ if root 2 eligible ⊕

k = pixels per em

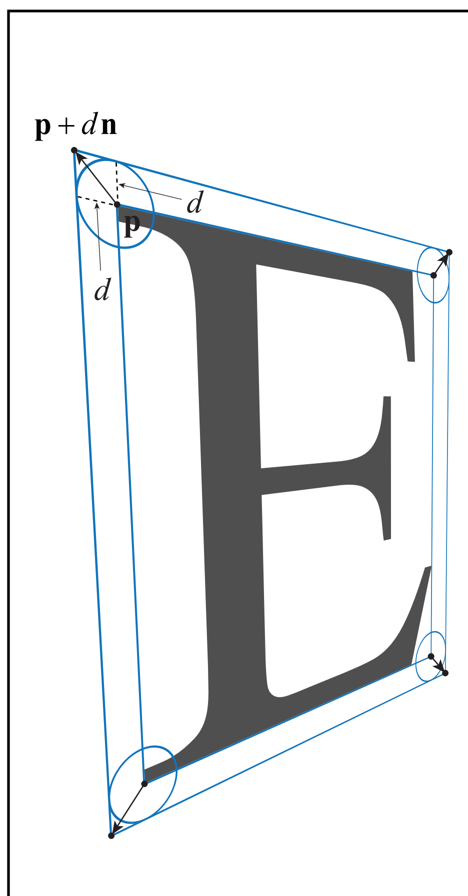
Geometry Bands



Bounding Polygons



Dynamic Dilation



$\mathbf{m}$ = transformation matrix from object space to clip space

w = viewport width $\qquad h$ = viewport height

d = object-space dilation distance

(p_x, p_y) = object-space vertex position

(n_x, n_y) = object-space vertex normal $\qquad \hat{\mathbf{n}} = \mathbf{n}/\|\mathbf{n}\|$

$$\Delta x = \frac{w}{2}\left[\frac{m_{00}(p_x + d\hat{n}_x) + m_{01}(p_y + d\hat{n}_y) + m_{03}}{m_{30}(p_x + d\hat{n}_x) + m_{31}(p_y + d\hat{n}_y) + m_{33}} - \frac{m_{00}p_x + m_{01}p_y + m_{03}}{m_{30}p_x + m_{31}p_y + m_{33}}\right]$$

$$\Delta y = \frac{h}{2}\left[\frac{m_{10}(p_x + d\hat{n}_x) + m_{11}(p_y + d\hat{n}_y) + m_{13}}{m_{30}(p_x + d\hat{n}_x) + m_{31}(p_y + d\hat{n}_y) + m_{33}} - \frac{m_{10}p_x + m_{11}p_y + m_{13}}{m_{30}p_x + m_{31}p_y + m_{33}}\right]$$

$$(\Delta x)^2 + (\Delta y)^2 = \frac{1}{4}$$ (half-pixel viewport-space dilation)

$$s = m_{30}p_x + m_{31}p_y + m_{33} \qquad t = m_{30}\hat{n}_x + m_{31}\hat{n}_y$$

$$u = w\left[s\left(m_{00}\hat{n}_x + m_{01}\hat{n}_y\right) - t\left(m_{00}p_x + m_{01}p_y + m_{03}\right)\right]$$

$$v = h\left[s\left(m_{10}\hat{n}_x + m_{11}\hat{n}_y\right) - t\left(m_{10}p_x + m_{11}p_y + m_{13}\right)\right]$$

$$d = \frac{s^3 t + s^2\sqrt{u^2 + v^2}}{u^2 + v^2 - s^2 t^2}$$